MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

EEM2036 – ENGINEERING MATHEMATICS III

(ME/RE/TE)

8 MARCH 2019

3.00 P.M.– 5.00 P.M.

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This exam paper consists of 9 **printed pages** (including cover page and appendix) with **four (4) questions** only.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

4. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

(a) For the shaded region of Figure Q1, determine

(i) the double integral formula that represents the area of the shaded region and

(ii) the area of the shaded region.

[4+6 marks]

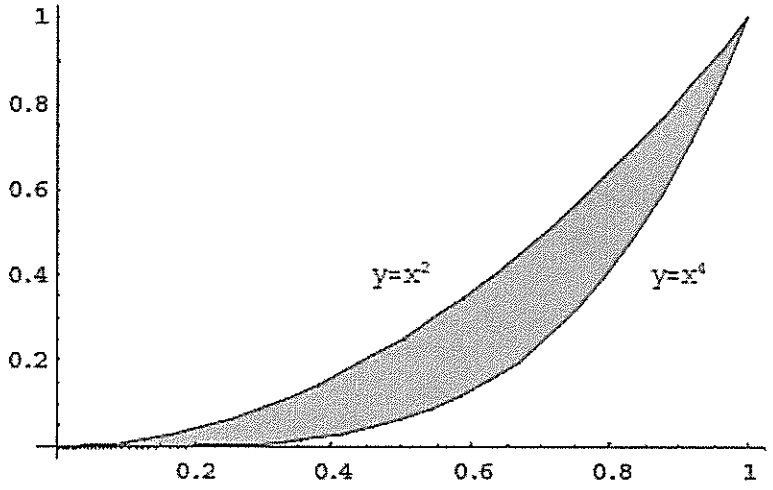

Figure Q1

(b) Let G be solid in the first octant bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the coordinate planes. In each part evaluate

$$\iiint_G xyz \; dV$$

(i) using rectangular coordinates.

(ii) using cylindrical coordinates

(iii) using spherical coordinates.

[5+5+5 marks]

Continued .....

Question 2

a) Suppose that

$$\vec{F}(x,y,z) = (x^2 + y)\hat{i} + z^2\hat{j} + (e^y - z)\hat{k}$$

i) Find the divergence of $\vec{F}(x,y,z)$.

ii) Hence, by using the Divergence Theorem, evaluate $\iint_\sigma \vec{F}\cdot\hat{n}dS$ where σ is the surface of the rectangular solid bounded by the coordinate planes and the planes $x = 3$, $y = 1$ and $z = 2$.

[2+6 marks]

b) Let R be the region between circles $x^2 + y^2 = 36$ and $x^2 + y^2 = 1$. Sketch the region and evaluate the line integral $\int_C 2xydy + (x^2 + 2x)dy$ using the Green's Theorem

[8 marks]

c) Use the Stoke's Theorem to evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F}(x,y,z) = -y^2\hat{i} + z\hat{j} + x\hat{k}$ and C be the positive oriented triangle lying on the plane $2x + 2y + z = 6$ bounded by the coordinate planes.

[9 marks]

Continued .....

Question 3

a) Solve the following the linear equation using the Gaussian Elimination method.

$$2x + 3y - z = 1$$
$$3x + 5y + 2z = 8$$
$$-x + 2y + 3z = 1$$

[12 marks]

b) A system of differential equation is given as follows:

$$x_1' = 5x_1 + 8x_2 + 16x_3$$
$$x_2' = 4x_1 + x_2 + 8x_3$$
$$x_3' = -4x_1 - 4x_2 - 11x_3$$

Find the general solution for the above system and a solution that satisfying the following boundary conditions

$$x_1(0) = x_2(0) = x_3(0) = 1.$$

[13 marks]

Continued .....

Question 4

a) By using the Simpson's rule, approximate $\int_1^3 \ln(x)dx$ with 4 intervals. Give your answer correct to 4 decimal places.

[8 marks]

b) Find a positive root of $e^x + x^2 - 10 = 0$ by using Newton-Raphson method with initial value as 2. Take the termination criterion as $|x_n - x_{n-1}| < \varepsilon$ with $\varepsilon = 1 \times 10^{-4}$.

[9 marks]

c) The matrix, A

$$A = \begin{bmatrix} -4 & 14 \\ -5 & 13 \end{bmatrix}$$

has eigenvalues $\lambda_1 = 6$ and $\lambda_2 = 3$. Perform five iterations of power method with initial vector $\mathbf{V} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ to approximate the dominant eigenvalue.

[8 marks]

Continued ....

APPENDIX

TABLE OF FORMULAS

1. The nth Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, ..., x_k](x - x_0) \cdots (x - x_{k-1}) .$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1)...(x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0) .$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n) .$$

Continued...

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h)-f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x)-f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left|\frac{h}{2} f''(c_x)\right|.$$

7. Central difference formula

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

with the error term

$$\left|\frac{h^2}{6} f^{(3)}(c_x)\right|.$$

8. Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}[f(a)+f(b)]$$

for some ξ in (a, b) and $h = b - a$, with the error term is $\left|\frac{h^3 f''(\xi)}{12}\right|$.

9. Composite Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a)+f(b)+2\sum_{j=1}^{n-1} f(x_j)\right]$$

for some ξ in (a, b) and $h = \frac{b-a}{n}$, with the error term is $\left|\frac{(b-a)h^2 f''(\xi)}{12}\right|$.

Continued ...

10. Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

for some ξ in (a, b) and $h=\frac{b-a}{2}$, with the error term $\left|\frac{h^5}{90}f^{(4)}(\xi)\right|$.

11. Composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j})+4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1})+f(b)\right]$$

for some ξ in (a, b) and $h=\frac{b-a}{n}$, with the error term $\left|\frac{(b-a)h^4}{180}f^{(4)}(\xi)\right|$.

12. Newton-Raphson's method

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}, \qquad n=0,1,2,...$$

13. Euler's method

$$y_{i+1}=y_i+hf(x_i,y_i)$$

with local error $\frac{h^2}{2}Y''(\xi_i)$ for some ξ_i in (x_i, x_{i+1}).

14. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1}=y_i+\tfrac{1}{2}(k_1+k_2)$$
$$k_1=hf(x_i,y_i)$$
$$k_2=hf(x_i+h,y_i+k_1)$$

Continued ...

15. Runge Kutta method of order four

$$
\begin{aligned}
k_1 &= hf(x_i, y_i), \\
k_2 &= hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1\right), \\
k_3 &= hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2\right), \\
k_4 &= hf(x_{i+1}, y_i + k_3), \\
y_{i+1} &= y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).
\end{aligned}
$$

End of Paper